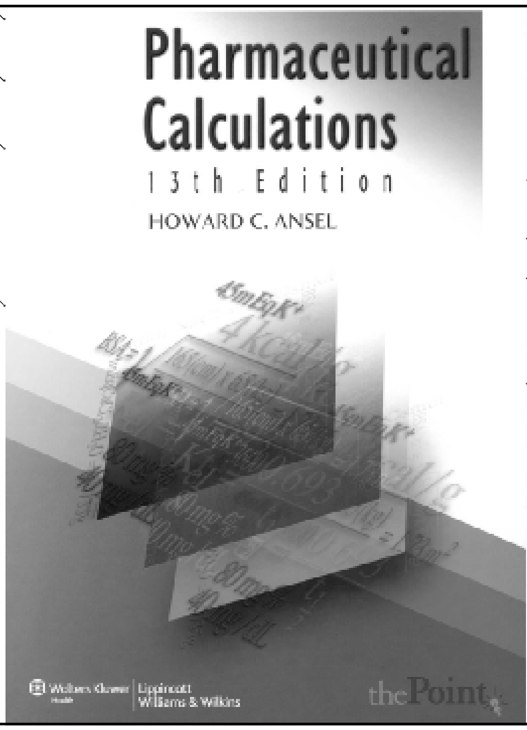
Fundamentals of Pharmaceutical Calculations

藥學計算的基礎



Scope of Pharmaceutical Calculations

The use of calculations in pharmacy is varied and broad-based. It encompasses calculations performed by pharmacists in traditional as well as in specialized practice settings and within operational and research areas in industry, academia, and government. In the broad context, the scope of pharmaceutical calculations includes computations related to:

- chemical and physical properties of drug substances and pharmaceutical ingredients;
- biological activity and rates of drug absorption, bodily distribution, metabolism and excretion (pharmacokinetics);
- statistical data from basic research and clinical drug studies;
- pharmaceutical product development and formulation;
- prescriptions and medication orders including drug dosage, dosage regimens, and patient compliance;
- pharmacoeconomics; and other areas.

Step-wise Approach pharmaceutical calculations

Step 1. Take the time necessary to carefully read and thoughtfully consider the problem *prior to* engaging in computations. An understanding of the purpose or goal of the problem and the types of calculations that are required will provide the needed direction and confidence.

Step 2. Estimate the dimension of the answer in both quantity and units of measure (e.g., milligrams) to satisfy the requirements of the problem. A section in Chapter 1 provides techniques for *estimation*.

Step 3. Perform the necessary calculations using the appropriate method both for efficiency and understanding. For some, this might require a step-wise approach whereas others may be capable of combining several arithmetic steps into one. Mathematical equations should be used only after the underlying principles of the equation are understood.

Step 4. Before assuming that an answer is correct, the problem should be read again and all calculations checked. In pharmacy practice, pharmacists are encouraged to have a professional colleague check all calculations prior to completing and dispensing a prescription or medication order. Further, if the process involves components to be weighed or measured, these procedures should be double-checked as well.

Step 5. Consider the *reasonableness* of the answer in terms of the numerical value, including the proper position of a decimal point, and the units of measure.

Objectives

- 分數，小數，百分比換算及計算應用
  - 利用指數計算
  - 應用比率及百分比方法解決問題
  - 應用維分析(dimentional analysis)方法解題
  - 理解有效數字

NUMBERS(數) AND NUMERALS(數字)

- A number is a total quantity, or amount, of units.
- A numeral is a word or sign, or a group of words or signs, expressing a number.
- For example, 3, 6, and 48 are Arabic numerals expressing numbers that are, respectively, 3 times, 6 times, and 48 times the unit.

NUMBERS(數) AND NUMERALS(數字)

- There are many symbols used in mathematics and science that provide instructions for a specific calculation or that indicate relative value.
- Some of the common symbols of arithmetic are presented in Table 1.1.

TABLE 1.1 SOME ARITHMETIC SYMBOLS USED IN PHARMACY[a]

SYMBOL	MEANING
%	percent; parts per hundred
‰	per mil; parts per thousand
+	plus; add; or positive
−	minus; subtract; or negative
±	add or subtract; plus or minus; positive or negative; expression of range, error, or tolerance
÷	divided by
/	divided by
×	times; multiply by
<	value on left is less than value on right (e.g., 5<6)
=	is equal to; equals
>	value on left is greater than value on right (e.g., 6>5)
≠	is not equal to; does not equal
≈	is approximately equal to
≡	is equivalent to
≤	value on left is less than or equal to value on right
≥	value on left is greater than or equal to value on right
.	decimal point
,	decimal marker (comma)
:	ratio symbol (e.g., *a*:*b*)
::	proportion symbol (e.g., *a*:*b*::*c*:*d*)
∝	varies as; is proportional to

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::	proportion symbol (e.g., *a*:*b*::*c*:*d*)
∝	varies as; is proportional to
x^2	x squared
x^3	x cubed

[a] Table adapted from *Barron's Mathematics Study Dictionary* by Frank Tapson with the permission of the author. Many other symbols (either letters or signs) are used in pharmacy, as in the metric and apothecaries' systems of weights and measures, in statistics, in pharmacokinetics, in prescription writing, in physical pharmacy, and in other areas. Many of these symbols are included and defined elsewhere in this text.

KINDS OF NUMBERS

- In arithmetic, the science of calculating with positive, real numbers,
- a number is usually
  1) a natural or whole number, or integer, such as 549;
  2) a fraction, or subdivision of a whole number, such as 4/7.
  3) a mixed number, consisting of a whole number plus a fraction, such as $3\frac{7}{8}$.

- A number such as 4, 8, or 12, taken by itself, without application to anything concrete, is called an abstract or pure number (抽象數或虛數).
  - merely designates how many times the unit 1 is contained in it, without implying that anything else is being counted or measured.
- An abstract number may be added to, subtracted from, multiplied by, or divided by any other abstract number.

- A number that designates a quantity of objects or units of measure, such as 4 grams, 8 milliliters, or 12 ounces, is called a *concrete or denominate number* (具體數或單位數).
  - designates the total quantity of whatever has been measured.
  - may be added to or subtracted from any other number of the same denomination,
  - may be multiplied or divided only by a pure number.

- The result of any of these operations is always a number of the same denomination.
- Examples:
  - Ø10 grams + 5 grams = 15 grams
  - Ø10.0 milliliters — 5 milliliters = 5 milliliters
  - Ø300 milligrams X 2 = 600 milligrams
  - Ø12 ounces ÷3=4 ounces

- When we apparently multiply or divide a denominate number by a number of different denomination, we are in fact using the multiplier or divisor as an abstract number.
  - for example, 1 ounce costs 5 ∈ (美分) and we want to find the cost of 12 ounces, we do not multiply .5 ∈ by 12 ounces, but by the abstract number 12.

ARABIC NUMERALS

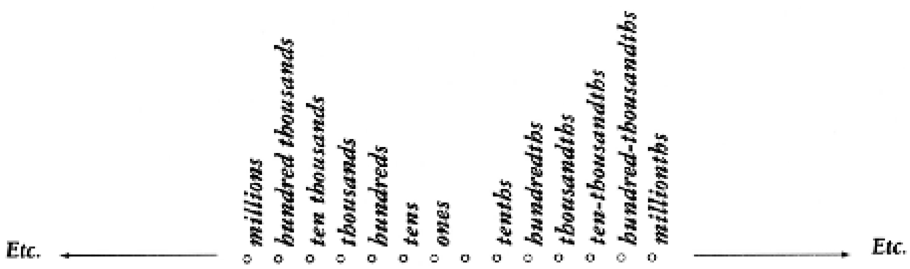
- is properly called a decimal system.
- With only 10 figures—a zero and nine digits (1,2,3,4,5,6,7,8,9)
- different values are assigned to the digits according to the place they occupy in a row.
- The central place in the row is usually identified by a sign placed to its right called the decimal point.

ARABIC NUMERALS

- Any digit occupying this place expresses its own value—in other words, a certain number of ones.
- The former value of a digit is increased 10-fold each time it moves one place to the left, and, conversely, its value is one-tenth of its preceding value each time it moves one place to the right.

- These 10 figures:
  - serve all our needs in dealing with positive integers,
  - are adequate for expressing fractions, negative numbers, and irrational (無理) and imaginary numbers (虛數).

Scheme of the decimal system:



- The total value of any number expressed in the Arabic (decimal) system is the sum of the values of its digits as determined by their position.
- Example: 5,083.623 means:

  5,000.000 or 5 thousands

  + 000.000 plus 0 hundreds

  + 080.000 plus 8 tens

  + 003.000 plus 3 ones

  + 000.600 plus 6 tenths

  + 000.020 plus 2 hundredths

  + 000.003 plus 3 thousandths

ROMAN NUMERALS

- The Roman system of notation expresses a fairly large range of numbers by the use of a few letters of the alphabet in a simple "positional" notation indicating adding to or subtracting from a succession of bases extending from 1 through 5, 10, 50, 100, and 500 to 1000.
- Roman numerals merely record quantities: they are of no use in computation.

- To express quantities in the Roman system, eight letters of fixed values are used:

ss	=	½
I or i	=	1
V or v	=	5
X or x	=	10
L or l	=	50
C or c	=	100
D or d	=	500
M or m	=	1000

- Other quantities are expressed by combining these letters by the general rule
  - when the second of two letters has a value equal to or smaller than that of the first, their values are to be added;
  - when the second has a value greater than that of the first, the smaller is to be subtracted from the larger.

Rule 1. Two or more letters express a quantity that is the sum of their values if they are successively equal or smaller in value:

ii	=	2	xv	=	15	lxxvii	=	77	dv	=	505	mc	=	1100
iii	=	3	xx	=	20	lxxxvii	=	88	dx	=	510	md	=	1500
vi	=	6	xxii	=	22	ci	=	101	dl	=	550	mdclxvi	=	1666
vii	=	7	xxxiii	=	33	cv	=	105	dc	=	600	mdcclxxvi	=	1776
viii	=	8	li	=	51	cx	=	110	mi	=	1001	mm	=	2000
xi	=	11	lv	=	55	cl	=	150	mv	=	1005	mmv	=	2005
xii	=	12	lx	=	60	cc	=	200	mx	=	1010			
xiii	=	13	lxvi	=	66	di	=	501	ml	=	1050			

Rule 2. Two or more letters express a quantity that is the sum of the values remaining after the value of each smaller letter has been subtracted from that of a following greater letter:

- dates are customarily expressed in capitals.
- Roman numerals are used in pharmacy only occasionally on prescriptions:
  - to designate the number of dosage units prescribed (e.g., capsules no. C),
  - to indicate the quantity of medication to be administered (e.g., teaspoonfuls ii),
  - the common or apothecaries' systems of measurement are used (e.g., grains iv)

iv	=	4	xxiv	=	24	xliv	=	44	cdi	=	401	cm	=	900
ix	=	9	xxxix	=	39	xc	=	90	cdxl	=	440	cmxcix	=	999
xiv	=	14	xl	=	40	xcix	=	99	cdxliv	=	444	MCDXCII	=	1492
xix	=	19	xli	=	41	cd	=	400	cdxc	=	490	MMIV	=	2004

Examples:

ii = 2	xxx = 30
iii = 3	xiii = 13
iv = 4	xiv = 14
vi = 6	xviii = 18
vii = 7	xix = 19
ix = 9	ci = 101

cxi = 111	lxxxviii = 88
dl = 550	xciv = 94
mv = 1005	cdxliv = 444
cd = 400	cdxc = 490
mc = 1100	cmxcix = 999
cm = 900	MCDXCI = 1492

COMMON AND DECIMAL FRACTIONS

- The arithmetic of pharmacy requires facility in the handling of common fractions and decimal fractions.
- The following brief review of certain principles and rules should be helpful,
- The practice problems should provide a means of gaining accuracy and speed in their manipulation.

Common Fractions

- A number in the form 1/8, 3/16.
- Its denominator, or second or lower figure, always indicates the number of aliquot parts into which 1 is divided;
- its numerator, or first or upper figure, specifies the number of those parts with which we are concerned.
- The value of a fraction is the quotient (i.e., the result of dividing one number by another) when its numerator is divided by its denominator.
  - If the numerator is smaller than the denominator, the fraction is called proper, and its value is less than 1.
  - If the numerator and denominator are alike, its value is 1.
  - If the numerator is larger than the denominator, the fraction is called improper, and its value is greater than 1.

- Two principles must be understood by anyone attempting to calculate with common fractions.
  1. multiplying the numerator increases the value of a fraction, and multiplying the denominator decreases the value, but when both numerator and denominator are multiplied by the same number, the value does not change.

$$\frac{2}{7}=\frac{3X2}{3X7}=\frac{6}{21}$$

- allows us to reduce two or more fractions to a common denomination when necessary.
- We usually want the lowest common denominator, which is the smallest number divisible by all the other given denominators.
  - ***Example: Reduce the fractions ¾, 4/5, and 1/3 to a common denomination***.

$$\left\{\begin{array}{l} \frac{3}{4}=\frac{15X3}{15X4}=\frac{45}{60} \\ \frac{4}{5}=\frac{12X4}{12X5}=\frac{48}{60} \\ \frac{1}{3}=\frac{20X1}{20X3}=\frac{20}{60} \end{array}\right\} answers$$

2. dividing the numerator decreases the value of a fraction, and dividing the denominator increases the value, but *when both numerator and denominator are divided by the same number, the value does not change*.

$$\frac{6}{21}=\frac{6\div 3}{21\div 3}=\frac{2}{7}$$

- allows us to reduce an unwieldy fraction to more convenient lower terms, either at any time during a series of calculations or when recording a final result.

- To reduce a fraction to its ***lowest terms***, divide both the numerator and the denominator by the largest common divisor(最大公約數).
  - ***Example: Reduce 36/2880 to its lowest terms.***

$$\frac{36}{2880}=\frac{36\div 36}{2880\div 36}=\frac{1}{80}, answer.$$

Rule 1. Before performing any arithmetical operation involving fractions, reduce every mixed number to an improper fraction.

- multiply the integer, or whole number, by the denominator of the fractional remainder, add the numerator, and write the result over the denominator.
- For example, before attempting to multiply 3/4 by 1 1/5, first reduce the l 1/5 to an improper fraction:

$$1\frac{1}{5}=\frac{(1X5)+1}{5}=\frac{6}{5}$$

Rule 2. When performing an operation involving a fraction and a whole number, express (or at least visualize) the whole number as a fraction having 1 for its denominator.

- Think of 3, as 3/1, 42 as 42/1, and so on.
- This visualization is desirable
  - when a fraction is subtracted from a whole number,
  - when a fraction is divided by a whole number.

Adding Fractions

- reduce them to a common denomination,
- add the numerators,
- write the sum over the common denominator.
  - If whole and mixed numbers are involved, the safest (although not the quickest) procedure is to apply Rules 1 and 2.
  - If the sum is an improper fraction, you may want to reduce it to a mixed number.

- *Example: In preparing batches of a formula, a pharmacist used 1/4 ounce, l/12 ounce, 1/8 ounce, and 1/6 ounce of a chemical. Calculate the total quantity of chemical used.*
- The lowest common denominator of the fractions is 24.

$$\frac{6+2+3+4}{24} ounce = \frac{15}{24} ounce$$

$$= \frac{5}{8} ounce, answer.$$

Subtracting Fractions

- Ø reduce them to a common denomination,
- Ø subtract, and write the difference over the common denominator.
  - If a whole or mixed number is involved, first apply Rule 1 or 2.
  - If the difference is an improper fraction, you may want to reduce it to a mixed number.

Examples: *A hospitalized patient received 7/12 liter of a prescribed intravenous infusion. If he had not received the final 1/8 liter, what fraction of a liter would he have received?*

$$\frac{7}{12}-\frac{1}{8}=\frac{14}{24}-\frac{3}{24}=\frac{11}{24}\,liter, answer$$

- *If 3 fl. oz. of a liquid mixture are to contain 1/24 fl. oz. of ingredient A, ¼ fl. oz. of ingredient B, and 1/3 oz. of ingredient C, how many fluidounces of ingredient D are required?*

$$\frac{1}{24}+\frac{1}{4}+\frac{1}{3}=\frac{1+6+8}{24}=\frac{15}{24}=\frac{5}{8}\,fl.oz.$$

$$\frac{3}{1}-\frac{5}{8}=\frac{24-5}{8}=\frac{19}{8}\,fl.oz.=2\frac{3}{8}\,fl.oz., answer.$$

Multiplying Fractions

- multiply the numerators
- write the product over the product of the denominators.
  - If either is a mixed number, first apply Rule 1.
  - If the multiplier is a whole number, simply multiply the numerator of the fraction and write the product over the denominator.

- Example: *If the adult dose of a medication is 2 teaspoonfuls, calculate the dose for a child if it is 1/4 of the adult dose.*

$$\frac{2\,tsp}{1} X \frac{1}{4} = \frac{2}{4} = \frac{1}{2} tsp., answer.$$

Dividing Fractions

- The reciprocal of a number is 1 divided by the number.
- For example, *the reciprocal of 3 is 1/3*.
- In general, when a is a fraction, its reciprocal is 1/a.
  - the reciprocal of 1/4 is 4/1 or 4,
  - the reciprocal of $2\frac{1}{2}$ or 5/2 is 2/5.

- if the fraction 3/4 is interpreted as meaning
  - 3 divided by 4,
  - dividing by 4 is exactly the same as multiplying by the reciprocal of 4, or ¼.
- This method of handling division when fractions are involved is called the reciprocal method,
- To divide by a fraction, then, simply invert its terms and multiply.
  - When a fraction is to be divided by a whole number, first interpret the whole number as a fraction, having 1 for its denominator, invert to get its reciprocal, and multiply.

- *Examples:* ***If 1/2 ounce is divided into 4 equal parts, how much will each part contain?***

$$\frac{1}{2} oz. \div \frac{4}{1} = \frac{1}{2} X \frac{1}{4} = \frac{1}{8} oz.., answer$$

A manufacturer wishes to prepare samples of an ointment in sealed foil envelopes, each containing 1/32 ounce of ointment. How many samples may be prepared from 1 pound (16 ounces) of ointment?

$$\frac{16}{1} \div \frac{1}{32} = \frac{16}{1} X \frac{32}{1} = 512 samples, answer.$$

If a child's dose of a cough syrup is ¾ teaspoonful and represents 1/4 of the adult dose, what is the adult dose?

$$\frac{3}{4}\,tsp. \div \frac{1}{4} = \frac{3}{4}\,tsp. X\ \frac{4}{1} = 3\ tsp., answer.$$

Decimal Fractions

- A fraction with a denominator of 10 or any power of 10 is called a decimal fraction, or simply a decimal.
- The denominator of a decimal fraction is never written, because the decimal point indicates the place value of the numerals.
- The numerator and the decimal point are sufficient to express the fraction.
- Therefore, 1/10 is written 0.1, 45/100 is written 0.45, and 65/1000 is written 0.065.

Familiar operations worth recalling

1. moving the decimal point one place to the right multiplies a number by 10, two places to the right multiplies it by 100, and so on.
2. moving the point one place to the left divides a number by 10, two places to the left it divides it by 100, and so on.
3. A decimal fraction may be changed to a common fraction by writing the numerator over the denominator and (if desired) reducing to lowest terms:

   0.125=125/1000=1/8
4. A common fraction may be changed to a decimal by dividing the numerator by the denominator (note that the result may be a repeating or endless decimal fraction):

   3/8=3÷8=0.375

   1/3=1÷3=0.3333…

Percent

- its corresponding sign,% , mean "in a hundred."
- 50 percent (50%) means 50 parts in each one hundred of the same item.
- Common fractions may be converted to percent by dividing the numerator by the denominator and multiplying by 100%.
- ***Example: Convert 3/8 to percent.***

$$\frac{3}{8} X\, 100\% = 37.5\%$$

- Decimal fractions may be converted to percent by multiplying by 100%.
- Example: *Convert 0.125 to percent.*
  0.125 X 100% = 12.5%, answer.

TABLE 1.2 EQUIVALENCIES OF COMMON FRACTIONS, DECIMAL FRACTIONS, AND PERCENT

COMMON FRACTION	DECIMAL FRACTION	PERCENT (%)	COMMON FRACTION	DECIMAL FRACTION	PERCENT (%)
1/1000	0.001	0.1	1/5	0.2	20
1/500	0.002	0.2	1/4	0.25	25
1/100	0.01	1	1/3	0.333	33.3
1/50	0.02	2	3/8	0.375	37.5
1/40	0.025	2.5	2/5	0.4	40
1/30	0.033	3.3	1/2	0.5	50
1/25	0.04	4	3/5	0.6	60
1/15	0.067	6.7	5/8	0.625	62.5
1/10	0.1	10	2/3	0.667	66.7
1/9	0.111	11.1	3/4	0.75	75
1/8	0.125	12.5	4/5	0.8	80
1/7	0.143	14.3	7/8	0.875	87.5
1/6	0.167	16.7	8/9	0.889	88.9

EXPONENTIAL NOTATION

- Many physical and chemical measurements deal with either very large or very small numbers.
  - Because it often is difficult to handle conveniently numbers of such magnitude in performing even the simplest arithmetic operations, it is best to use exponential notation or powers of 10 to express them.
- we may express 121 as 1.21×10^{2}, 1210 as 1.21×10^{3}, and 1,210,000 as 1.21×10^{6}.
- we may express 0.0121 as 1.21×10^{-2}, 0.00121 as 1.21×10^{-3}, and 0.00000121 as 1.21×10^{-6}.

- The first part is called the coefficient, customarily written with one figure to the left of the decimal point.
- The second part is the exponential factor or power of 10.
- The exponent represents the number of places that the decimal point has been moved—positive to the left and negative to the right—to form the exponential.
  - when we convert 19000 to 1.9×10^4, we move the decimal point 4 places to the left; hence the exponent 4.
  - when we convert 0.0000019 to 1.9×10^{-6}, we move the decimal point 6 places to the right; hence the negative exponent.

FUNDAMENTAL ARITHMETIC OPERATIONS WITH EXPONENTIALS

- In the *multiplication* of exponentials,
  - the exponents are added. For example, $10^2 \times 10^4 = 10^6$.
  - the coefficients are multiplied together in the usual manner,
- Examples:

$(2.5 \times 10^2) \times (2.5 \times 10^4) = 6.25 \times 10^6$, or 6.3×10^6

$(2.5 \times 10^2) \times (2.5 \times 10^{-4}) = 6.25 \times 10^{-2}$, or 6.3×10^{-2}

$(5.4 \times 10^2) \times (4.5 \times 10^3) = 24.3 \times 10^5 = 2.4 \times 10^6$

- In the *division* of exponentials,
  - the exponents are subtracted.
    - For example, $10^2 \div 10^5 = 10^{-3}$.
  - the coefficients are divided in the usual way.
- Examples:

  $(7.5 \text{ X } 10^5) \div (2.5 \text{ X } 10^3) = 3.0 \text{ X } 10^2$

  $(7.5 \text{ X } 10^{-4}) \div (2.5 \text{ X } 10^6) = 3.0 \text{ X } 10^{-10}$

  $(2.8 \text{ X } 10^{-2}) \div (8.0 \text{ X } 10^{-6}) = 0.35 \text{ X } 10^4 = 3.5 \text{ X } 10^3$

- In the *addition and subtraction* of exponentials,
  - the expressions must be changed (by moving the decimal points) to forms having any common power of 10,
  - the coefficients are added or subtracted.
- The result should be rounded off to the number of decimal places contained in the least precise component, and it should be expressed with only one figure to the left of the decimal point.
- Examples:

  $(1.4 \text{ X } 10^4) + (5.1 \text{ X } 10^3)$

  $(1.4 \text{ X } 10^4) - (5.1 \text{ X } 10^3)$

  $(9.83 \text{ X } 10^3) + (4.1 \text{ X } 10^1) + (2.6 \text{ X } 10^3)$

COMMON LOGARITHMIC NOTATION

- *Every number is expressed simply as a power of* 10—not with absolute precision, but with sufficient accuracy for any given purpose—and we may multiply any two numbers so expressed, or divide one by the other, by the simple process of adding or subtracting their exponents.
- The exponent that indicates to what power 10 must be raised to equal approximately a given number is called the common logarithm of that number.
- The logarithm of 10 or of any integral power of 10 is always a positive or negative integer:

log 10 (or 1×10^{1}) == 1

log 100 (or 1×10^{2}) = 2

log 1000 (or 1×10^{3}) = 3

and so on; and

log 1 (or $1 \times 10^{\circ}$) = 0

log 0.1 (or 1×10^{-1}) = -1

log 0.01 (or 1×10^{-2}) = -2

- If these were the only numbers in existence, no table of logarithms would be needed; for a given number, say 1,000,000 (or 1×10^6),
  - If we know the system, we can readily supply its logarithm: 6; or, given the logarithm 6, we can readily reconstruct the number it represents: 1,000,000.

- Any number not in the 10's series must contain a certain excess over some power of 10—as 150 contains 10^2 plus an excess of 50.
  - the logarithm of such a number always consists of a positive or negative whole-numbered exponent plus a positive decimal-fraction exponent (carried to as many decimal places as suit our purposes).
  - As it turns out, the power of 10 that approximates 150 (or 1.5×10^2) is $10^{2.1761}$, and therefore log 150 = 2.1761.

- The *whole-number exponent* is called the *characteristic*.
  - It accounts for the integral power of 10 contained in the given number and hence serves to locate the decimal point in that number.
  - If a number is given in ordinary notation, you can find the characteristic by converting it to exponential notation, in which the characteristic appears as a power of 10.

- The *decimal-fraction exponent* is called the *mantissa*, which you can find in a table of logarithms.
  - The mantissa represents the *significant figures* in a given number, regardless of the location of the decimal point.
- In other words, given the sequence 610, a four-place table will tell you the mantissa is 7853, whether the number is 61.0, or 6.10, or 0.00610.

ously:

Characteristics | Mantissas

log 6.10 (or 6.10×10^0) $= 0.7853$

log 61.0 (or 6.10×10^1) $= 1.7853$

log 610 (or 6.10×10^2) $= 2.7853$

log 6100 (or 6.10×10^3) $= 3.7853$

and so on; and

log 0.610 (or 6.10×10^{-1}) $= \bar{1}.7853$

log 0.0610 (or 6.10×10^{-2}) $= \bar{2}.7853$

log 0.00610 (or 6.10×10^{-3}) $= \bar{3}.7853$

and so on. Note that putting the minus sign *over* the characteristic indicates it alone is negative, and that the mantissa, as always, is positive.

NATURAL LOGARITHMS

- The base of the *natural or Naperian* system of logarithms is e, which is the irrational number 2.71828 ....
- When it is necessary to change from a natural logarithm to a common logarithm, the computation may be performed by using the following relationship:

$$\log_e n = 2.303 \log_{10} n$$

2.303 is the logarithm of 10 to the base 2.71828.

$\text{Log}_e 10 = 2.303$

USE OF LOGARITHM TABLES

- Logarithm tables give mantissas calculated to four-place, five-place accuracy, and upward, depending on the table and its purpose.
- A four-place table ensures an accuracy within 0.5 when we work with three-figure numbers.
- Table l has typical features.
  1. a column to the left and a row at the top to guide us in locating the mantissas of three-figure numbers;
  2. the four-place mantissas of all three-figure numbers,
  3. columns of proportional parts providing us with a quick means of calculating more accurate mantissas when given numbers of four-figure accuracy, a process called *interpolation*.

Table 1. Logarithms

Natural Numbers	0	1	2	3	4	5	6	7	8	9	Proportional Parts 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

(continued)

Table 1. Logarithms

Natural Numbers	0	1	2	3	4	5	6	7	8	9	Proportional Parts 1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Finding the Logarithm of a Number.

- determine the characteristic, then find the mantissa in the log table.
- *Find the log of 262.*

  $262 = 2.62 \times 10^2$

  By inspection of the ten factor, the characteristic = 2.

  To find the mantissa, focus attention on the digits 262.

  In the left-hand column in the log table, find 26; opposite it and in the column numbered 2 is the desired mantissa 0.4183. (The table omits the 0.)

  Therefore, log 262 = 2.4183, answer.

- *Find the log of 2627.*

  $2627 = 2.627 \text{ X } 10^3$

  By inspection of the ten factor, the characteristic = 3.

  In the left-hand column in the table, find 26; opposite it and in the column numbered 2, find the mantissa 0.4183; opposite 26 and in column 7 under proportional parts, find 11 (meaning 0.0011 but written without zeros) and add it to 0.4183 to obtain the desired mantissa 0.4194.

  Therefore, log 2627 = 3.4194, answer.

- *Find the log of 0.002627.*

  $0.002627 = 2.627 \text{ X } 10^{-3}$

  By inspection of the ten factor, the characteristic = $\underline{3}$.

  The mantissa is determined as in the preceding example.

  Therefore, log 0.002627 = $\underline{3}$.4194, answer.

Finding the Antilogarithm of a Logarithm

- When a problem is solved by logarithms, the result is expressed as the logarithm of the answer.
- Therefore, it is necessary to find the antilogarithm or the number corresponding to the logarithm.
- If the mantissa of a logarithm is known, its anti-logarithm can be found by a reverse reading of the log table.
- *Find the antilogarithm of the logarithm 1.7604.*

  The mantissa 0.7604 is found in the column numbered 6 opposite 57, and the resulting figure is 576.

  The characteristic is 1 and the required number is 5.76 X 10^1 or 57.6, answer.

- ***Find the antilogarithm of the logarithm 3.7607***.

  Because the mantissa 0.7607 is not found in the log table, interpolation must be used.

  In the log table, 0.7607 falls between 0.7604 and 0.7612; therefore, the resulting figure must be between 576 and .577.

  The given mantissa is 0.0003 (or 3 units) more than the mantissa 0.7604.

  Therefore, opposite 0.7604, find 3 in column 4 of proportional parts.

  The required figure is 5764.

  The characteristic is 3 and the required number is 5.764 X 10^3 = 5764. answer.

Table A.I.1. Logarithms

Natural Numbers	0	1	2	3	4	5	6	7	8	9	Proportional Parts								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

(continued)

Table A.I.1. (Continued)

Natural Numbers	0	1	2	3	4	5	6	7	8	9	Proportional Parts								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9026	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

SOME LOGARITHMIC COMPUTATIONS

- As shown in the first of the subsequent examples, when a negative number is "added", it is actually subtracted;
- As shown in the third example, when a negative number is "subtracted," it is actually added.
- The fourth example shows the curious but consistent fact, when subtracting one logarithm from another, that if you borrow from a negative characteristic (as 1 is borrowed from the -1 of the minuend), you increase the value of the negative characteristic (as the -1 becomes -2, which is canceled out when the 2 of the subtrahend is “subtracted” from it).

- ***Multiply (5.25 X 10^3) by (8.92 X 10^{-6}) by (7.56 X 10^5).***

log (5.25X 10^3) =	3.7202
log (8.92X 10^{-6}) =	$\bar{6}$.9504
log (7.56X 10^5) =	5.8785
Total:	4.5491

Antilogarithm of 4.5491 = 3.541 X 10^4 = 35410, or (retaining only three significant figures), 35400, answer.

- ***Divide 29600 by 5.544.***

  29600 = 2.96×10^4

  5.544 = 5.544×10^0

  log (2.96×10^4) = 4.4713

  log (5.544×10^0) = 0.7438

  Difference: 3.7275

  Antilogarithm of 3.7275 = 5.34×10^3 = 5340, answer.

- ***Divide 7500 by 0.627.***

  7500 = 7.50×10^3

  0.627 = 6.27×10^{-1}

  log (7.50×10^3) = 3.8751

  log (6.27×10^{-1})= $\underline{1}.7973$

  Difference: 4.0778

  Anrilogarichm of 4.0778 = 1.196×10^4 = 11960, or (retaining only three significance figures), 12000, answer.

- ***Divide 0.191 by 0.0452.***

 $0.191 = 1.91 \times 10^{-1}$

 $0.0452 = 4.52 \times 10^{-2}$

 $\log (1.91 \times 10^{-1}) = \underline{1}.2810$

 $\log (4.52 \times 10^{-2}) = \underline{2}.6551$

 Difference: 0.6259

 Anrilogarichm of $0.6259 = 4.226 \times 10^{0} =$ 4.226, or (retaining only three significant figures), 4.23, answer.

- ***Find the value of***

$$\frac{(4.54 \times 10^{6}) \times (3.25 \times 10^{3})}{(1.21 \times 10^{8})}$$

 $\log (4.54 \times 10^{6}) = 6.6571$

 $\log (3.25 \times 10^{3}) = 3.5119$

 Total: 10.1690 = log of numerator

 $\log (1.21 \times 10^{8}) =$ 8.0828 = log of denominator

 Difference: 2.0862

 Antilogarithm of 2.0862 = 1.219 or $1.22 \times 10^{2} = 122$, answer.

RATIO, PROPORTION, AND VARIATION

Ratio

- The relative magnitude of two like quantities is called their ratio.
- Ratio is sometimes defined as the quotient of two like numbers.
- To avoid losing sight of the fact that two quantities are being compared, this quotient is always expressed as an operation, not as a result: in other words, it is expressed as a fraction, and the fraction is interpreted as indicating the operation of dividing the numerator by the denominator.
- A ratio presents us with the concept of a common fraction as expressing the relation of its two numbers.

- The ratio of 20 and 10 is not expressed as 2 (that is, the quotient of 20 divided by 10), but as the fraction 20/10.
- When the fraction 1/2 is to be interpreted as a ratio, it is traditionally written 1:2, and it is read not as one-half but as 1 to 2.
- All the rules governing common fractions equally apply to a ratio.
- If the two terms of a ratio are multiplied or are divided by the same number, the value is unchanged.
  - eg: the ratio 20:4 or 20/4 has a value of 5; if both terms are divided by 2, the ratio becomes 10:2 or 10/2, again the value of 5.

- The terms of a ratio must be of the same kind, for the value of a ratio is an abstract number expressing how many times greater or smaller the first term (or numerator) is than the second term (or denominator).
- The terms may be abstract numbers, or they may be concrete numbers of the same denomination.
  - we can have a ratio of 20 to 4(20/4) or 20 grams to 4 grams (20 grams/4 grams).

- When two ratios have the same value, they are equivalent.
  - the *product of the numerator of the one and the denominator of the other always equals the product of the denominator of the one and the numerator of the other, i.e., the cross products are equal:*

    ∵ 2/4 = 4/8

    2 X 8 (or 16) = 4 X 4 (or 16)

$$\because \frac{2}{4} = \frac{4}{8}$$

- *if two ratios are equal, their reciprocals are equal*:

  ∵ 2/4 = 4/8, then 4/2 = 8/4
- *the numerator of the one fraction equals the product of its denominator and the other fraction*:

  If 6/15 = 2/5

  then 6 = 15 X 2/5 = 6,

  and 2 = 5 X 6/15 = 2.
- *the denominator of the one equals the quotient of its numerator divided by the other fraction*:

  15 = 6 ÷ 2/5 (or 6 X 5/2) = 15,

  and 5 = 2 ÷ 6/15 (or 2 X 15/6) = 5.

Proportion

- A proportion is the expression of the equality of two ratios. It may be written in any one of three standard forms:
  1. a:b = c:d
  2. a :b :: c :d
  3. a/b=c/d
- Each of these expressions is read: a is to b as c is to d, and a and d are called the *extremes* (meaning "outer members") and b and c the *means* ("middle members").

- In any proportion, the product of the extremes is equal to the product of the means.
- If the missing term is a mean, it will be the product of the extremes divided by the given mean,
  - if it is an extreme, it will be the product of the means divided by the given extreme.

$$If\ \frac{a}{b}=\frac{c}{d}, then$$

$$a=\frac{bc}{d}, b=\frac{ad}{c}, c=\frac{ad}{b}, and\ d=\frac{bc}{a}$$

- *given any three terms of a proportion, by appeal to the facts set forth above, we may easily calculate the value of the fourth.*
- Because the missing fourth is usually the desired answer, proportion takes us to it without any intermediate steps.

- *Examples:*
  1. *If 3 tablets contain 975 milligrams of aspirin, how many milligrams should be contained in 12 tablets?*

$$\frac{3(tablets)}{12(tablets)} = \frac{975(mg)}{x(mg)}$$

$$x = \frac{12X975}{3} mg = 3900mg, answer.$$

Examples:

If 3 tablets contain 975 milligrams of aspirin, how many tablets should contain 3900 milligrams?

$$\frac{3(tablets)}{x(tablets)} = \frac{975(mg)}{3900(mg)}$$

$$x = \frac{12X975}{3} mg = 3900mg, answer.$$

- *the means or the extremes may be interchanged without destroying the validity of the equation.*

$$\frac{3(tablets)}{12(tablets)} = \frac{975(mg)}{3900(mg)}$$

$$\frac{3(tablets)}{975(mg)} = \frac{12(tablets)}{3900(mg)}$$

$$\frac{3900(mg)}{12(tablets)} = \frac{975(mg)}{3(tablets)}$$

$$\frac{3900(mg)}{975(mg)} = \frac{3(tablets)}{12(tablets)}$$

- *ratios should express the relationship of denominate numbers of the same kind.* In many problems, the quantities given must be reduced or converted to a common denomination before we can proceed with the solution.
- **Proportions** need not contain whole numbers. If common or decimal fractions are supplied in the data, they may be included in the proportion without changing the method.

1. Two fractions having a **common denominator** are **directly proportional to their numerators**.
2. Two fractions having a **common numerator** are **inversely proportional to their denominators**.

$$\frac{\frac{60}{100}}{\frac{50}{100}} = \frac{60}{50}$$

$$\text{Proof}: \frac{60}{100} \div \frac{50}{100} = \frac{60}{100} X \frac{100}{50} = \frac{60}{50}$$

$$\frac{\frac{2}{3}}{\frac{2}{7}} = \frac{7}{3}$$

$$\text{Proof}: 2/3 \div 2/7 = 2/3 \times \frac{7}{2} = 7/3$$

- Example: ***If 30 milliliters represent 1/6 of the volume of a prescription, how many mililiters will represent 1/4 of the volume?***

$$\frac{\frac{1}{6}(volume)}{\frac{1}{4}(volume)} = \frac{30(mL)}{x(mL)}$$

$$Or: \frac{4}{6} = \frac{30}{x}(mL)$$

$$x = \frac{6 \times 30}{4} mL = 45mL, answer.$$

Variation

- Most pharmaceutical calculations deal with:
  - **simple, direct relationships**: twice the cause, double the effect.
  - **inverse relationships**: twice the cause, half the effect, as when you decrease the strength of a solution by increasing the amount of diluent.
- *If 10 pints of a 5 % solution are diluted to 40 pints, what is the percentage strength of the dilution*?

$$\frac{10(pints)}{40(pints)} = \frac{x(\%)}{5(\%)}$$

$$x = \frac{10 \times 5}{40}\% = 1.25\%, answer.$$

CALCULATIONS CAPSULE

Ratio and Proportion

- A *ratio* expresses the relative magnitude of two like quantities (e.g., 1:2, expressed as "1 to 2.")
- A *proportion* expresses the equality of two ratios (e.g., 1:2 = 2:4).
- The four terms of a proportion are stated as:

$$a{:}b = c{:}d, \text{ or, } a{:}b :: c{:}d, \text{ or } \frac{a}{b} = \frac{c}{d}$$

  and expressed as "*a* is to *b* as *c* is to *d*."
- Given three of the four terms of a proportion, the value of the fourth, or missing, term may be calculated.
- The ratio-and-proportion method is a useful tool in solving many pharmaceutical calculation problems.

Dimensional analysis

Factor analysis
Factor-label method
Unit-factor method

Step 1. Identify the given quantity and its unit of measurement.
Step 2. Identify the wanted unit of the answer.
Step 3. Establish the *unit path* (to go from the given quantity and unit to the arithmetic answer in the wanted unit), and identify the conversion factors needed. This might include:
(a) a conversion factor for the given quantity and unit, and/or
(b) a conversion factor to arrive at the wanted unit of the answer.
Step 4. Set up the ratios in the unit path such that cancellation of units of measurement in the numerators and denominators will retain only the desired unit of the answer.
Step 5. Perform the computation by multiplying the numerators, multiplying the denominators, and dividing the product of the numerators by the product of the denominators.

CALCULATIONS CAPSULE

Dimensional Analysis

- An alternative method to ratio and proportion in solving pharmaceutical calculation problems.
- The method involves the logical sequencing and placement of a series of ratios to consolidate multiple arithmetic steps into a single equation.
- By applying select conversion factors in the equation—some as reciprocals—unwanted units of measure cancel out, leaving the arithmetic result and desired unit.
- Dimensional analysis scheme:

Unit Path

Given Quantity	Conversion Factor for Given Quantity	Conversion Factor for Wanted Quantity	Conversion Computation		Wanted Quantity
				=	

How many fluidounces (fl. Oz.) are there in 2.5 liters (L)?

Unit Path

Given Quantity	Conversion Factor for Given Quantity	Conversion Factor for Wanted Quantity	Conversion Computation		Wanted Quantity
2.5 L	1000 mL	1 fl. oz.		=	
	1 L	29.57 mL			

Note: The unit path is set up such that all units of measurement will cancel out except for the unit wanted in the answer, *fluidounces*, which is placed in the numerator.

- *Examples: How many fluidounces are in 2.5 liters if there are 1000 milliliters in 1 liter and 29.57 milliliters in 1 fluidounce?*

$$Step\ 1:$$
$$\frac{2.5L}{1.0L} = \frac{x\,mL}{1000\,mL}$$
$$Step\ 2:$$
$$\frac{29.57\,mL}{1\,fl.oz.} = \frac{2500\,mL}{x\,fl.oz.}$$
$$x = \frac{2500}{29.57} = 84.5\,fl.oz., answer.$$

$$\frac{1\,fl.oz.}{29.57\,mL} \times \frac{1000\,mL}{1\,L} \times 2.5\,L = 84.5\,fl.oz., answer.$$

- *A medication order calls for 1000 milliliters of a dextrose intravenous infusion to be administered over an 8-hour period. Using an intravenous administration set that delivers 10 drops/milliliter, how many drops per minute should be delivered to the patient?*

$$8\,hrs = 480\,mins$$
$$\frac{1000\,mL}{x\,mL} = \frac{480\,mins}{1\,min}$$
$$x = 2.1\,mL/min$$

$$\frac{2.1\,mL}{x\,drops} = \frac{1\,mL}{10\,drops}$$
$$x = 21\,drops/min, answer.$$

$$\frac{10\,drops}{1\,mL} \times \frac{1000\,mL}{480\,mins} = 21\,drops/min, answer.$$

SIGNIFICANT FIGURES

CALCULATIONS CAPSULE

Significant Figures

- Digits other than zero are significant.
- A zero between digits is significant.
- Final zeros after a decimal point are significant.
- Zeros used only to show the location of the decimal point are not significant.

SIGNIFICANT FIGURES

- When we record a *measurement*, the last figure to the right must be taken to be an ***approximation***, an admission that the limit of possible precision or of necessary accuracy has been reached, and that any further figures to the right would be **non-significant**.
- We should learn to interpret a denominate number like **325 grams** as follows: The *3 means 300 grams*, neither more nor less, and the 2 means *exactly 20 grams more*; but the final 5 means *approximately 5 grams more*, i.e., *5 grams plus or minus some fraction of a gram*.
- Whether this fraction is, for a given purpose, negligible depends on **how precisely** the quantity was (or is to be) weighed.

- **Significant figures**, then, are consecutive figures that express the value of a denominate number **accurately enough** for a given purpose.
- The accuracy varies with the number of significant figures, which are all absolute in value except the **last**, and this is properly called **uncertain**.
  - **Two-figure accuracy** is liable to a deviation as high as **5%** from the theoretic absolute measurement.
    - For example, if a substance is reported to weigh 10 grams to the nearest gram, its actual weight may be anything between **9.5 and 10.5 grams**.
  - **Three-figure accuracy** is liable to a deviation as high as **0.5%;**
  - **four-figure accuracy** may deviate **0.05%;**
  - **five-figure accuracy**, **0.005%.**

The interpretation of zero may be summed up as follows:

1. Any zero between digits is significant.
2. Initial zeros to the left of the first digit are never significant; they are included merely to show the location of the decimal point and thus give place value to the digits that follow.
3. One or more final zeros to the right of the decimal point may be taken to be significant.

Examples:

Assuming that the following numbers are all denominate:

1. In *12.5*, there are *three* significant figures; in *1.256*, *four* significant figures; and in *102.56*, *five* significant figures.
2. In *0.5*, there is *one* significant figure. The digit *5* tells us how many *tenths* we have. The nonsignificant *0* simply calls attention to the decimal point.
3. In *0.05*, there is still only *one* significant figure, as there is in *0.005*.
4. In *0.65*, there are *two* significant figures, and likewise *two* in *0.065* and *0.0065*.
5. In *0.0605*, there are *three* significant figures. The first *0* calls attention to the decimal point, the second *0* shows the number of places to the right of the decimal point occupied by the remaining figures, and the third *0* significantly contributes to the value of the number. In *0.06050*, there are *four* significant figures, because the final *0* also contributes to the value of the number.

Assuming that the following numbers are all denominate:

1. 12.5: three significant figures;
2. 1.256: four significant figures;
3. 102.56: five significant figures.
4. 0.5: one significant figure. The digit 5 tells us how many tenths we have. The nonsignificant 0 simply **calls attention to the decimal point**.
5. 0.05: is still only one significant figure, and again in 0.005.
6. 0.65: two significant figures, and likewise two in 0.065 and 0.0065.

Assuming that the following numbers are all denominate:

5. 0.0605: three significant figures.
   - The first 0 calls attention to the decimal point;
   - the second 0 shows the **number of places** to the right of the decimal point occupied by the remaining figures;
   - the third 0 significantly contributes to the value of the number.
- 0.06050: four significant figures, because the final 0 also contributes to the value of the number.

6. 20000: five significant figures;
7. 20000 ± 50 or, to express the same quantity another way, 20000 to the nearest 100, contains only three significant figures.

- one of the factors determining the degree of approximation to perfect measurement is the **precision of the instrument** used. It would be incorrect to claim:
  - 7.76 milliliters had been measured in a graduate calibrated in units of 1 milliliter,
  - 25.562 grams had been weighed on a balance sensitive to 1/10 grain,

1. If a substance weighs 0.06 gram,
   1. according to a balance sensitive to 0.001 gram, we may record the weight as 0.060 gram.
   2. But if the balance is sensitive only to 0.01 gram, the value should be recorded as 0.06 gram, and a record of 0.060 gram would be invalid.
2. When recording a length of **10 millimeters** found by use of an instrument accurate to **0.1** millimeter, the value may be recorded as **10.0** millimeters.
3. If a volume of **5 milliliters** is measured with an instrument calibrated in **tenths** of a milliliter, the volume may be recorded as **5.0** milliliters.

- We must clearly distinguish ***significant figures***, from ***decimal places***.
  - When recording a measurement, the number of decimal places we include indicates *the degree of precision with which the measurement has been made,*
  - whereas the number of significant figures retained indicates the *degree of accuracy* that is sufficient for a given purpose.
- a value "**correct to (so-many) decimal places** should never confuse with the expression "**correct to (so-many) significant figures**.

Examples

1. If the value of 27.625978 is rounded off to five decimal places, it is written 27.62598; but when this value is rounded off to five significant figures, it is written 27.626.
2. The value 54.3265, when rounded off to 54.3, is precise to **one decimal place** but it is accurate to **three significant figures**.

Rules for rounding

1. when **recording a measurement**, retain as many figures as will give only one uncertain figure. The **uncertain figure** will sometimes represent an estimate **between graduations on a scale**.
    1. if you use a ruler calibrated in centimeters, you might record a measurement as approximately **11.3** centimeters, but not as approximately 11.32 centimeters. Because the 3 is uncertain, no other figure should follow it.

2. when **eliminating superfluous figures** in the result of a calculation, **add 1 to the last figure** retained if the following figure is **5 or more**.
    1. 2.43 may be rounded off to 2.4, but 2.46 should be rounded off to 2.5.
    2. if a number like 2.597 is rounded off to three significant figures, the 1 added to the 9 makes 10, and 0 should be recorded, for it is significant: 2.60.

3. when adding or subtracting approximate numbers, include only as many decimal places as are in the number with the **least decimal places**.

- *Example: Add these approximate weights: 162.4 grams, 0.489 gram, 0.1875 gram, and 120.78 grams.*

Incorrect	***Correct***
162.4	*162.4*
0.489	*0.5*
0.1875	*0.2*
+120.78	*+ 120.8*
283.8565, answer.	*283.9, answer.*

- It is important to note that in filling a prescription, the pharmacist must **assume that the physician means** each quantity to be measured with the *same degree of precision*. Hence, if we add these quantities taken from a prescription:

  5.5 grams
  0.01 gram
  0.005 gram

- we must not round off the total to one decimal place. Rather we must retain **at least three decimal places** in the total by interpreting the given quantities to mean **5.500** grams, **0.010** gram, and **0.005** gram.
- When **greater precision** is required, we may interpret the given quantities to mean **5.5000, 0.0100**, and **0.0050**, etc.

4. *when* ***multiplying or dividing*** *one approximate number by another approximate number,* ***round off the component with the greater number of significant figures to the number contained in the component having fewer significant figures****. Retain no more significant figures in the product or quotient than in the number with the least significant figures.*

Example: Multiply 1.65370 grams by 0.26.

- 1.65370 grams is rounded off to 1.65 grams
- 1.65 grams X 0.26 = 0.4290 or 0.43 gram, answer.

- When multiplying or dividing with denominate numbers taken from a **prescription or official formula**, ***assuming that each quantity*** is meant to be measured with the same degree of accuracy,
- we must interpret each quantity as having at least as many **significant figures** as appear in the quantity containing the **greatest number of significant figures**.
  - if the quantities 0.25 gram, 0.5 gram, and 5 grams are included in a prescription,
  - should be interpreted as 0.25 gram, 0.50 gram, and 5.0 grams for purposes of multiplication or division (as when we enlarge or reduce a formula),
  - results should be rounded off to contain **two significant figures**.
- **multiplication and division**: concerned with the number of significant figures,
- **addition and subtraction**: the number of decimal places is important.

5. When multiplying or dividing an **approximate number by an absolute number**, **round off the result to the same number of significant figures** as in the approximate number.

Example: If a patient has taken 96 doses, each containing 2.54 mg of active ingredient, how many milligrams of the active ingredient has he taken in all?

Ans: 2.54 mg
X 96
1524
2286
243.84 or 244 mg, answer.

ESTIMATION

- One of the best checks of the **reasonableness** of a numeric computation.
- If we arrive at a wrong answer by using a wrong method, mechanical final verification of our figuring may not reveal the error.
- But an absurd result, such as occurs when the **decimal point is put in the wrong place**, will not likely slip past if we check it against a preliminary estimation of what the result should be.

- Because it is imperative that pharmacists ensure the accuracy of their calculations by every possible means, pharmacy students are urged to adopt **estimation** as one of those means.
- Proficiency in estimating **comes only from constant practice**.
- Pharmacy students are urged to acquire the habit of estimating the answer to every problem encountered before attempting to solve it.
- Estimation serves not only as a means for **judging the reasonableness of the final result**, but also as a **guide in the solution of the problem**.

- Checking the accuracy of every calculation, of course, such as by adding a column first upward and then downward, is important.
- The student should follow this invariable procedure: (1) estimate, (2) compute, (3) check.
  - First, the numbers given in a problem are **mentally rounded off to slightly larger or smaller numbers** containing fewer significant figures; e.g., 59 would be rounded off to 60, and 732 to 700.
  - Then, the required computations are performed, as far as possible mentally, and the result, although known to be somewhat greater or smaller than the exact answer, is close enough to serve as an estimate.

- No set rules for estimating can be given to cover all the computations in arithmetic.
- But examples can illustrate some of the methods that can be used.
- One way to obtain a reasonable estimate of the total is first to add the figures in the **leftmost column**.
  - The neglected remaining figures of each number are equally likely to express more or less than one-half the value of a unit of the order we have just added,
  - hence to the sum of the leftmost column is added 1/2 for every number—or 1 for every two numbers—in the column.

- *Examples: Add the following numbers: 7428, 3652, 1327, 4605, 2791, and 4490.*
- Estimation: The figures in the thousands column add up to 21000, and with each number on the average contributing 500 more, or every pair 1000 more, we get 21000 + 3000 =24000, estimated answer.
- Calculation:

  7428
  3652
  1327
  4605
  2791
  4490
  ______
  24293, answer.

- *Add the following numbers: 2556, 449, 337, 1572.*
- Estimation: The figures of the thousands column add up to 3000, and with each pair of numbers contributing approximately another 1000, we get 3000 + 2000 = 5000, estimated answer.
- Calculation:

```
 2556
  449
  337
 1572
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 4914, answer.
```

- In multiplication, the product of the two leftmost digits plus a sufficient number of zeros to give the right place value serves as a fair estimate. The number of zeros supplied must equal the total number of all discarded figures to the left of the decimal point.

- Approximation to the correct answer is closer if the discarded figures are used to round off the value of those retained.
- *Examples: Multiply 612 by 415,*
- Estimation: 4 X 6 = 24, and because we discarded four figures, four zeros must be supplied, giving 240,000, estimated answer.
- Calculation:

612
X 413
1836
612
2448
252756, answer.

- *Multiply 2889 by 209.*
- Estimation: The given numbers round off to 3000 and 200. 3 X 2 = 6, and supplying five zeros we get 600,000, estimated answer.
- Calculation;

2889
X 209
26001
5778
603801, answer.

- When the multiplier is a decimal fraction, the possibility of error is reduced if we first convert it to a common fraction of approximately the same place.
- *Examples: Multiply 41.76 by 20.3.*
  - Estimate: 42 X 20 = 840.
- *Multiply 730.5 by 321.*
  - Estimate: 700 X 300 = 210,000.
- *Multiply 314.2 by 0.18.*
  - Estimate: Because 0.18 or 18/100 lies between 1/6 and 1/5 the answer will lie between 50 and 60.
- *Multiply 48.16 by 0.072.*
  - Estimate: 7/100 equals about 1/15, and 1/15 of 48 is about 3.

- In division, the given numbers may be rounded off to convenient approximations, but again care is needed to preserve the correct place values.
- Example: Divide 2456 by 5.91.
  - Estimate: The numbers may be rounded off to 2400 and 6. We may divide 24 by 6 mentally, but we must remember the two zeros substituted for the given 56 in 2456.
  - The estimated answer is 400.